Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Wednesday 20 January 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35381A

1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

2.
$$f(x) = x^3 + 2x^2 - 3x - 11$$

(*a*) Show that f(x) = 0 can be rearranged as

(2)

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α .

(*b*) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4 .

(3)

(*c*) Show that α = 2.057 correct to 3 decimal places.

(3)

---

3. (*a*) Express 5 cos x – 3 sin x in the form R cos(x + α), where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(*b*) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$, giving your answers to 2 decimal places.

(5)

---

4. (i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(5)

---

5. Sketch the graph of $y = \ln |x|$, stating the coordinates of any points of intersection with the axes.

(3)

6.

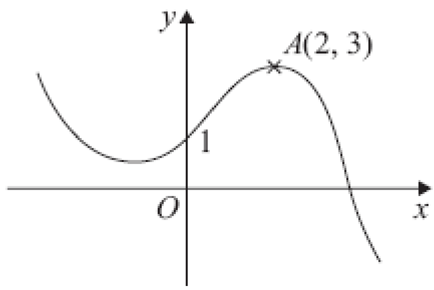


Figure 1

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = f(-x) + 1$,

(ii) $y = f(x + 2) + 3$,

(iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(9)

---

7. (*a*) By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = \mathrm{e}^{2x} \sec 3x$,

(*b*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\dfrac{\pi}{6} < x < \dfrac{\pi}{6}$, has a minimum turning point at (a, b).

(*c*) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

---

8. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(7)

---

9. (i) Find the exact solutions to the equations

(*a*) $\ln (3x - 7) = 5$,

(3)

(*b*) $3^x \mathrm{e}^{7x + 2} = 15$.

(5)

(ii) The functions f and g are defined by

$$\mathrm{f}(x) = \mathrm{e}^{2x} + 3, \qquad x \in \mathbb{R},$$

$$\mathrm{g}(x) = \ln (x - 1), \qquad x \in \mathbb{R}, \quad x > 1.$$

(*a*) Find f^{-1} and state its domain.

(4)

(*b*) Find fg and state its range.

(3)

TOTAL FOR PAPER: 75 MARKS

END

January 2010
6665 Core Mathematics C3
Mark Scheme

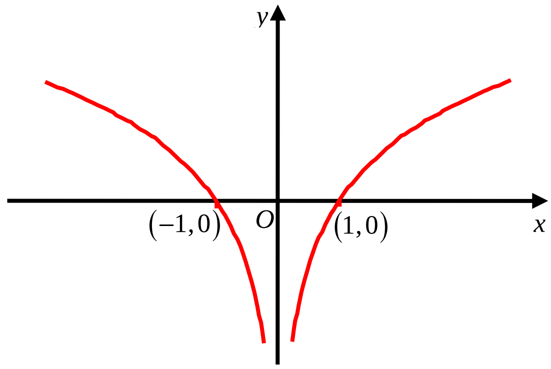
Question Number	Scheme		Marks
Q1	$\dfrac{x+1}{3x^2-3}-\dfrac{1}{3x+1}$		
	$=\dfrac{x+1}{3(x^2-1)}-\dfrac{1}{3x+1}$		
	$=\dfrac{x+1}{3(x+1)(x-1)}-\dfrac{1}{3x+1}$	$x^2-1 \to (x+1)(x-1)$ or $3x^2-3 \to (x+1)(3x-3)$ or $3x^2-3 \to (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$=\dfrac{1}{3(x-1)}-\dfrac{1}{3x+1}$		
	$=\dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	Attempt to combine.	M1
	or $\dfrac{3x+1}{3(x-1)(3x+1)}-\dfrac{3(x-1)}{3(x-1)(3x+1)}$	Correct result.	A1
		Decide to award M1 here!!	**M1**
	$=\dfrac{4}{3(x-1)(3x+1)}$	Either $\dfrac{4}{3(x-1)(3x+1)}$ or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ or $\dfrac{4}{9x^2-6x-3}$	A1 **aef**
			[4]

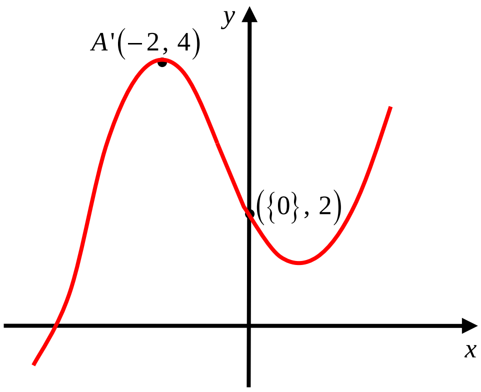
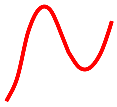
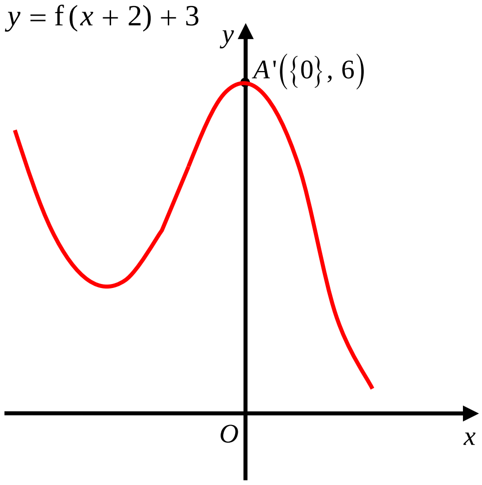
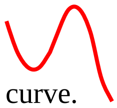
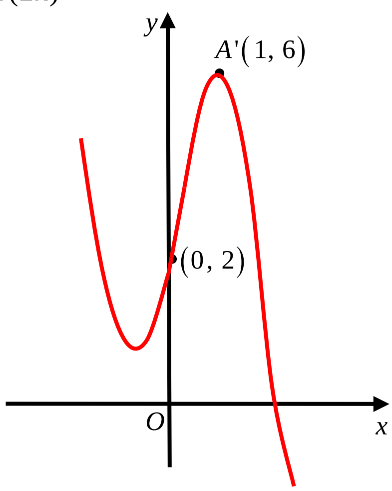
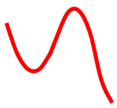
Question Number	Scheme		Marks
Q2	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \dfrac{3x+11}{x+2}$		
	$\Rightarrow x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 **AG**
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$ $\mathrm{f}(2.0575) = 0.0041401094...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
		any one value awrt 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

Question Number	Scheme		Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x+\alpha)$, $R > 0$, $0 < x < \frac{\pi}{2}$		
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$		
	$R = \sqrt{5^2+3^2}$; $= \sqrt{34}$ $\{= 5.83095..\}$	$R^2 = 5^2 + 3^2$ $\sqrt{34}$ or awrt 5.8	M1; A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$	M1
		$\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \dfrac{\pi}{\text{awrt } 5.8}$	A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$		
			(4)
(b)	$5\cos x - 3\sin x = 4$		
	$\sqrt{34}\cos(x+0.5404) = 4$		
	$\cos(x+0.5404) = \dfrac{4}{\sqrt{34}}$ $\{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \dfrac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\dfrac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$	awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c$ $\{= 5.468358...^c\}$	$2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$	awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$		
			(5)
			[9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

Question Number	Scheme		Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{ \begin{array}{ll} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{array} \right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{ \dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1) \right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $\{= \cos^2 y\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 **AG**
			(5)
			[9]

Question Number	Scheme		Marks
Q5	$y = \ln\lvert x\rvert$		
		Right-hand branch in quadrants 4 and 1. Correct shape.	B1
		Left-hand branch in quadrants 2 and 3. Correct shape.	B1
		Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
			(3)
			[3]

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$  	
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\},\ 2)$ or $A'(-2,\ 4)$	B1
	Both $(\{0\},\ 2)$ and $A'(-2,\ 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x+2) + 3$  	
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\},\ 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$  	
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\},\ 2)$ or $A'(1,\ 6)$	B1
	Both $(\{0\},\ 2)$ and $A'(1,\ 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q7 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$	Seen or implied	
		Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	A1 **isw**
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 **cao**
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme		Marks
Q8	$\operatorname{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$		
	Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\operatorname{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\operatorname{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad$ or $\quad \cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad$ or $\quad \cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad$ or $\quad \cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y-3)=2x$ $\Rightarrow \tfrac{1}{2}\ln(y-3)=x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x)=\underline{\tfrac{1}{2}\ln(x-3)}$	$\underline{\tfrac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\tfrac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain}>3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3$ $\{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range}>3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]